Distribution characteristic and combined optimization of maximum cogging torque of surface mounted few slots permanent-magnet machines

Ping Jin¹, Yujing Guo¹, Jianhu Yan²

1 School of Energy and Electrical Engineering, Hohai University, Nanjing 211110, P. R. China ² School of Automation, Nanjing University of Science and Technology, Nanjing 210094, P. R. China

This paper examines the maximum cogging torque (MCT) distribution characteristic of surface mounted few slots permanent magnet (SMFSPM) machines. The MCT distributions of SMFSPM machines with different pole-arc coefficients and different ratios of slot opening to slot-pitch are firstly analyzed based on the analytical Fourier series analysis with Maxwell stress tensor method and the finite element method (FEM), respectively. Right triangle distribution regions of the optimal MCT are determined by comparing the MCT distribution contour maps calculated by the two methods. Finally, a combined MCT optimization method for SMFSPM machines is proposed to result in a substantial saving in the optimization time.

*Index Terms***—maximum cogging torque, few slots permanent magnet machines, right triangle distribution.**

I. INTRODUCTION

Surface-mounted few slots permanent magnet (SMFSPM) machines with concentrated windings are attracting the interests of many researchers in recent years because of its short end winding and simple manufacturing process which allows the stator teeth to be manufactured separately to result in a high fill factor in the stator slots [1]-[2]. However, it is noted that improper optimization of the maximum cogging torque (MCT) may lead to serious noise and vibration, thereby undermining the machine's dynamic performances in a detrimental manner. Although it is feasible to use stator skewing to reduce the MCT, such approach is not very feasible in a high power SMFSPM machine due to its complicated construction. In order to reduce the MCT without skewing the stator, either the pole-arc coefficient or the ratio of slot opening to slot-pitch have been optimized based on numerical and analytical methods. A finite element method (FEM) based on a combination of electromagnetic field and circuit equations for cogging torque prediction is presented [3]. Based on an analytical technique using a relative permeance function, cogging torque can be predicted using either a one-dimensional (1-D) model or a two-dimensional (2-D) model [4]. However, pure analytical method is not accurate enough. Nonetheless, good design results can be obtained by employing the two methods simultaneously.

The purpose of this paper is to examine the MCT distribution characteristic of SMFSPM machines. The MCT distributions of SMFSPM machines with different pole-arc coefficients and ratios of slot opening to slot-pitch are analyzed based on analytical Fourier series analysis with Maxwell stress tensor method and by FEM. Right triangle distribution regions of the optimal MCT are determined by comparing the distribution contour maps obtained by the two methods. A combined MCT optimization method is finally proposed to optimize the MCT of SMFSPM machines with relatively low computer resources.

II. DISTRIBUTION OF MCT

A. Analytical method

Based on the analytical Fourier series analysis with Maxwell stress tensor method, its cogging torque can be expressed as

$$
T = \sum_{1}^{N_S} l_a \int_0^{\frac{(1-\delta_s)\pi}{N_S}} \frac{B_{a b 1}^2 - B_{a b 2}^2}{2\mu_0} (\alpha + R_s) d\alpha \tag{1}
$$

where R_s is the inner radius of stator, δ_s is the ratio of slot opening to slot-pitch, l_a is the effective axial length of machine, $B_{\alpha b1}$ and $B_{\alpha b2}$ are the two flux densities on the two sides of a tooth, which can also be analytical expressed.

Fig. 1 shows the MCT distribution contour map of the 6/8 SMFSPM machines based on the above analytical formulas, where the maximum MCT is chosen as a reference value and *Rr* , *Rm* and *Rs* are constants.

Fig. 1. The MCT distribution contour map based on the analytical analysis.

B. Numerical method

Fig. 2 shows the MCT distribution contour map of the typical 6/8 SMFSPM machine based on FEM. The hollow circular points in the contour map denote the optimal points. Comparing with the contour map as shown in Fig. 1, the optimal points seem to be rather randomly distributed. It can

be observed that the optimal δ_s values are not dependent on δ_p ; Neither is the optimal δ_p values depend on δ_s . However, when the optimal lines obtained by the analytical method and the optimal points obtained by the FEM are overlapped in one figure, the distribution of the optimal points looks more orderly.

Fig. 2. The MCT distribution contour map based on the FEM.

Fig. 3 gives an optimal MCT distribution by overlapping the optimal lines and the optimal points given by the two methods, respectively. It can be clearly seen that all optimal points are forming some right triangles. The optimal lines are generally constituting the two sides of these triangles.

Fig. 3. The right triangle distribution character.

For validating the universality of the right triangles distribution characteristic, three different SMFSPM machines are analyzed. The MCT distribution contour maps of the three machines are given. The conclusion is that all optimal points based on the FEM are in the right triangle regions obtained based on the analytical Fourier series analysis with Maxwell stress tensor method.

III. COMBINED OPTIMIZATION

Based on the distribution characteristic as observed, a combined MCT optimization method is proposed for SMFSPM machines. The searching space of the optimal MCT is limited in the right triangles which can be determined by analytical method. The program of the combined MCT optimization method is as follows:

1) calculate the optimal pole-arc coefficients and ratios of slot opening to slot-pitch, respectively, based on the analytical Fourier series analysis with the Maxwell stress tensor method;

2) determine the right-angled triangles containing the optimal points;

3) search for the optimal points in the right-angled triangles using FEM.

As there exists only two or three optimal points in each right-angled triangle, a local optimal algorithm, such as course and fine meshes, can be applied for searching for the optimal points. Table I gives a comparison of the optimal results and costing times with different methods when $\delta_s \in (0.5, 0.75)$ and $\delta_p \in (0.5, 0.75)$. The values of δ_s and δ_p are obtained by the analytical Fourier series analysis with Maxwell stress tensor method, FEM, the combined method, and the combined method with course and fine meshes respectively. The entries of Table I show that the combined method has a relatively high efficiency. Fig. 5 gives the cogging torque results of the 6/8 SMFSPM machine with optimal δ_s and δ_p obtained by the proposed methods.

TABLE I. COMPARISON OF THE OPTIMAL RESULTS AND COSTING TIMES

Methods	Optimal results (δ_n , δ_s)	Times
Analytical analysis	0.8739Nm(0.65, 0.75)	2 min
FEM		31hours
Combined method	0.2843 Nm $(0.68, 0.54)$	8 hours
Combined method with course and fine	0.2488 Nm $(0.67, 0.56)$ 0.1811 Nm $(0.62, 0.69)$	1.5 hours
meshes $*$		

*the size of course meshes is 0.03; the size of fine meshes is 0.01.

Fig. 5. The cogging torque results of the 6/8 SMFSPM machine.

ACKNOWLEDGMENT

This work was jointly supported by the NSFC (51407061) and the NSF of Jiangsu Province (BK20140854).

REFERENCES

- [1] A. J. Ortega, and L. Y. Xu, Fellow, Analytical prediction of torque ripple in surface-mounted permanent magnet motors due to manufacturing variations, IEEE Trans Energy Conver., vol. 31, no. 4, pp. 1634-1644, Dec. 2016.
- [2] Li Zhu, S. Z. Jiang, J. Z. Jiang, Z. Q. Zhu, and C. C. Chan, "A new simplex wave winding permanent-magnet brushless dc machine,' *IEEE Transactions on Magnetics*, vol. 47, no. 1, pp. 252-259, Jan. 2011.
- [3] S. J. Sung, S. J. Park, and G. H. Jang, "Cogging torque of brushless dc motors due to the interaction between the uneven magnetization of a permanent magnet and teeth curvature," *IEEE Transactions on Magnetics*, vol. 47, no. 7, pp. 1923-1928, Jul. 2011.
- [4] D. H. Wang, X. H. Wang, Y. B. Yang, and R. Zhang, "Optimization of magnetic pole shifting to reduce cogging torque in solid-rotor permanent-magnet synchronous motors," *IEEE Transactions on Magnetics*, vol. 46, no. 5, pp. 1228-1234, May. 2010.